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Generalized Main Sequence Stars:

One can generalize the concepts of main-sequence to consider chemically homogeneous stars with different chemical compositions. The properties of the star will change for different chemical composition (assuming the total mass is fixed).

As an example, lets consider homologous models with different chemical composition (but the same mass). The dependence of stellar parameters on the mean molecular weight μ is given by the following homology relations:

$$R \propto \mu^{m_R} \quad , \quad T \propto \mu^{n_T} \quad , \quad L \propto \mu^{m_L}$$

Where:

$$m_R = B \quad , \quad m_T = 1 - B \quad , \quad m_L = (4 + n + s) + B(3n - s)$$

$$B = \frac{\nu - s - (4+n)}{\nu - s + 3n + 4}$$

The constants n, s, ν are defined through the power-law relations on page 102. Note that ν enters stellar equations mainly through the ideal gas law and, more weakly, through the opacity.

For the CNO-cycle ($\nu = 15-18$) and electron-scattering opacity ($n = s = 0$), and using the ideal gas law, we get,

$m_L = 4.0$, $m_R = 0.58 - 0.63$, $m_T = 0.42 - 0.36$

This shows that the radius and temperature vary rather weakly with ν, whereas the luminosity is a strongly increasing function.

The slope in the H-R diagram will be:

$m_{HR} = \frac{1}{4}\left(1 - \frac{2m_R}{m_L}\right) \simeq 0.17$

We note that this is higher than the corresponding value $n_{HR} \simeq 0.12$, obtained for the main-sequence stars. This implies that the stars with the same mass but different values of μ will evolve below the main-sequence line in the H-R diagram.

A similar analysis can be performed to determine the dependence of the parameters of the star on the metal abundance Z.

The Helium Main Sequence:

We can describe a main sequence as any sequence of homogenous models in complete equilibrium, consisting mainly of a certain element that burns in the central region.

In this sense, the normal main sequence, treated before, is called the Hydrogen main sequence (H-MS).

The Helium main sequence (He-MS) contains chemically homogeneous equilibriom models that consist almost

Completely of He:

$X \simeq 0$, $Y \simeq 0.98$

We then find:

$$\nu \equiv \frac{\rho}{m_u n_{tot}} \qquad n_{tot} = 2\frac{\rho}{m_u}X + \frac{3}{4}\frac{\rho}{m_u}Y$$

$$\Rightarrow \nu = \frac{1}{2X+\frac{3}{4}Y} \simeq \boxed{1.36} \quad \text{He-MS}$$

This must be compared with ν for H-MS stars:

$X \simeq 0.7$, $Y \simeq 0.3 \Rightarrow \boxed{\nu \simeq 0.62}$ H-MS

In principle, one could imagine He-MS stars to be the descendents of perfectly mixed Hydrogen burning stars, although perfect mixing during evolution is very improbable. They may also represent the remnants of originally more massive stars that have developed a central Helium core (through Hydrogen burning) and then lost their Hydrogen-rich envelope.

For the same mass M, He-MS stars have a smaller radius than H-MS stars. This results in a larger central density ρ_c, which is expected in order to have Helium burning. They also have larger T_c, P_c and a larger luminosity. It turns out that $\Delta \log L \approx 1.33$ for $M = 10\,M_\odot$, which is in agreement with the prediction of homology relations $\Delta \log L = 4 \Delta \log \mu$. For $M = M_\odot$, however, it is found that $\Delta \log L \simeq 2.5$.

Since He-MS stars have a much higher central density, they reach the critical density $\rho_c \sim 10^5\ g\ cm^{-3}$ at a larger mass as compared with H-MS stars: $M \approx 0.3\,M_\odot$ vs $M \simeq 0.085\,M_\odot$. At this density the quantum-mechanical pressure from electrons become important, regardless of temperature T, and hence the sequence of stable stars terminate. This happens at $M \simeq 0.3\ M_\odot$ for He-MS stars as compared with $M \simeq 0.085\ M_\odot$ for H-MS stars.

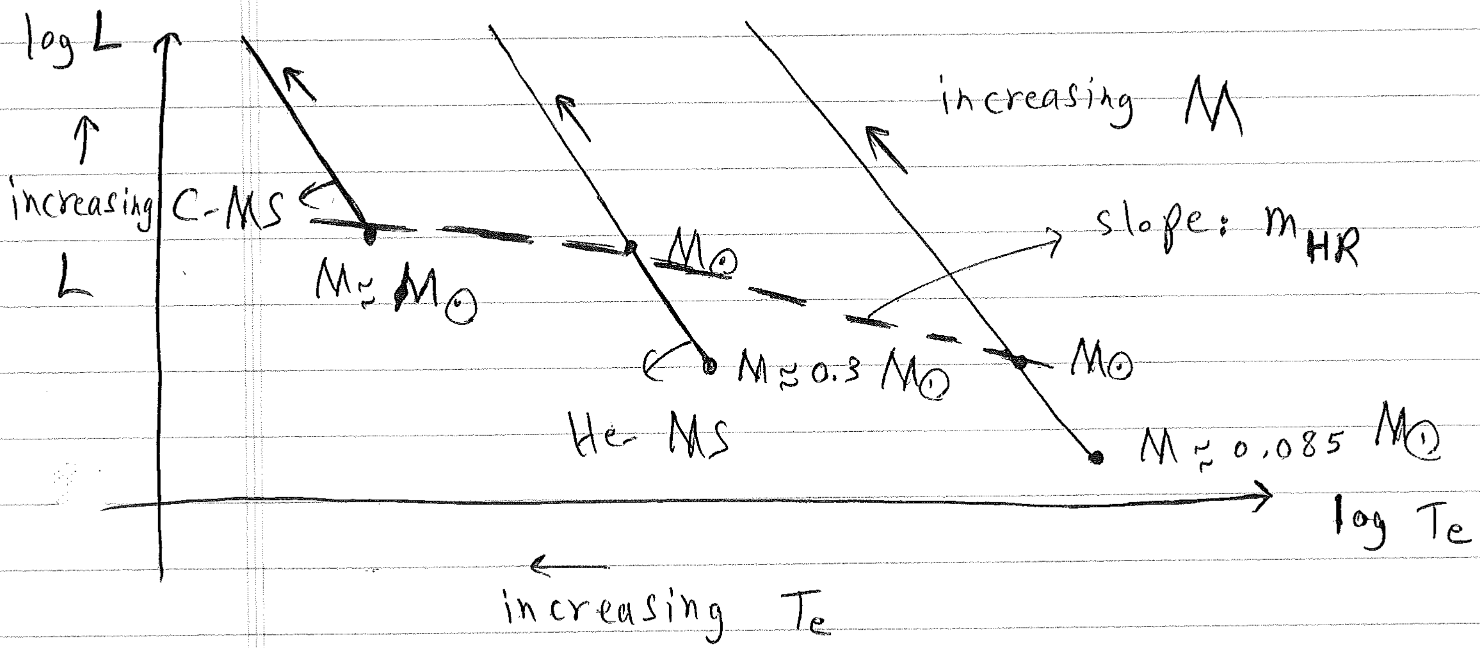
The Carbon Main Sequence:

A Carbon main-sequence (C-MS) star consists of homogeneous models in complete equilibrium that have central Carbon burning. Except for the usual admixture of few percent of heavy elements, the composition is either pure ^{12}C ($X_C \simeq 0.98$) or a mixture of ^{12}C and ^{16}O in equal amounts ($X_C \simeq X_O \simeq 0.49$). The mean molecular weight is:

$$\boxed{\mu = \frac{1}{\frac{7}{12}X_C + \frac{9}{16}X_O} \simeq 1.78} \qquad \text{C-MS}$$

The interior solutions of C-MS stars have similar properties to those of the He-MS stars. They have a smaller radius, larger luminosity, and larger central density than the He-MS stars (for the same mass). As a result, the sequence of stable C-MS stars terminates at an even larger mass in this case: $M \simeq M_\odot$ as compared with $M \simeq 0.3\, M_\odot$ (He-MS) and $M \simeq 0.085\, M_\odot$ (H-MS).

Schematically, the lines that represent the H-MS, He-MS and C-MS stars in the H-R diagram are as follows:



Generalized Main Sequence:

The logical next step is to drop the condition of chemical homogeneity as suggested by the chemical evolution in stars. The conversion of Hydrogen to Helium produces a central Helium region, while the outer envelope retains its Hydrogen rich mixture.

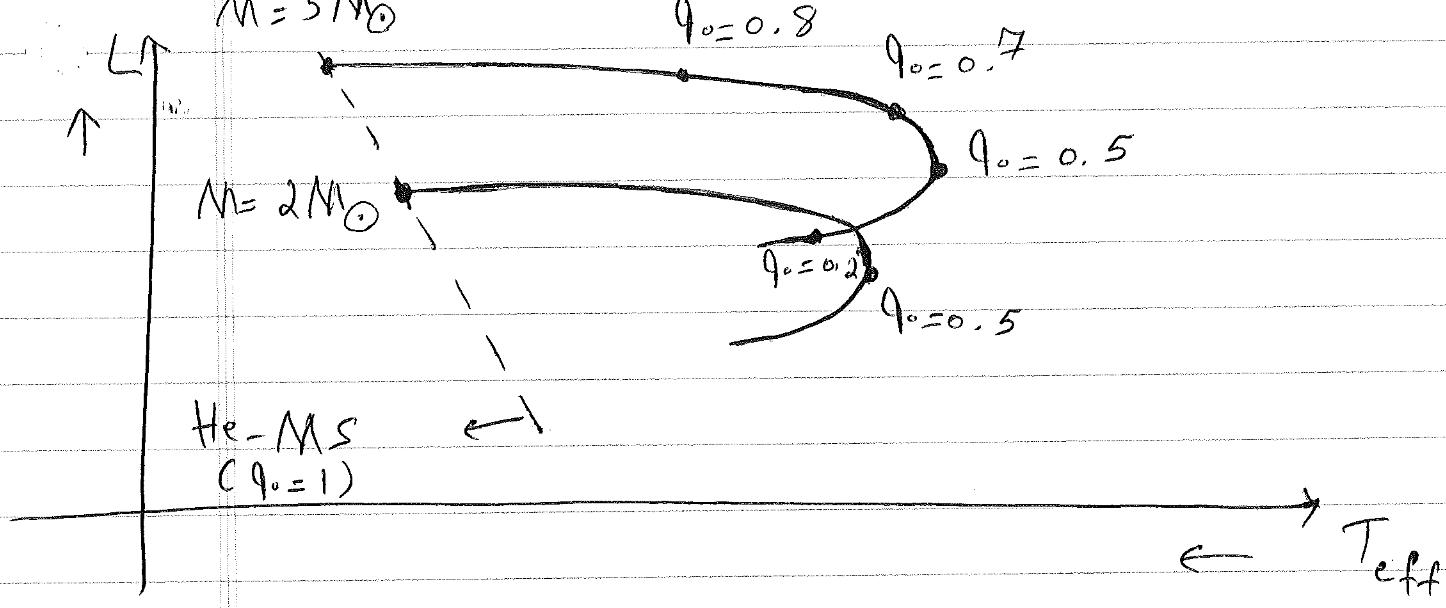
A simple model would be to consider a step profile. A central Helium Core of mass M_{He}, with mass fraction $q_0 = \frac{M_{He}}{M}$, is surrounded by an envelope of mass $(1-q_0) M$ with the usual Hydrogen rich mixture. The energy is is supplied by central Helium burning and an additional Hydrogen burning in a shell.

Each of these models is characterized by two parameters; M and q_0. The value of q_0 is not constant during the evolution: it can slowly increase because of the shell burning, and it can increase by mass loss from the surface. In the limits $q_0 = 1$ and $q_0 = 0$ we recover the He-MS and H-MS respectively.

For a fixed mass M, one can see how the position of the star in the H-R diagram changes with varying q_0. $q_0 = 1$ corresponds to a point on the line that represents He-MS stars. As q_0 decreases, the position of the star moves to the right along an approximately horizontal line (i.e. L remains almost constant, while T_e decreases). This can be understood as follows. With decreasing q_0, the luminosity of the core L_{He} decreases, while that of the Hydrogen shell source L_H increases. In consequence, the total luminosity $L = L_{He} + L_H$ can reamin almost constant.

The radius of the star increases with decreasing q_0 (recall that He-MS stars ($q_0 = 1$) with the same mass have a smaller radius than H-MS stars ($q_0 = 0$), varying q_0 will interpolate between the two cases). For a constant luminosity, a larger radius implies a smaller surface temperature T_e according to $L \propto R^2 T_e^4$.

This explains the motion of the star position to the right of L-T_e plane as q_0 decreases. However, there is a limiting line far to the right side of the H-R diagram beyond which the star position cannot move. The closest approach to it is found roughly for $q_0 = 0.5$, and for smaller values of q_0 the position of the star moves to the left of the H-R diagram. This is shown schematically in the following figure:



The solid lines connect models of the same stellar mass ($M = 5M_\odot$ and $M = 2M_\odot$) for different values of q_0. There seems to be a forbidden region in the right part of the H-R diagram, which is separated from the allowed region to the left by a limiting line. This is the so called "Hayashi Line" that we discuss next.

The Hayashi Line:

The Hayashi line is defined as the locus in the H-R diagram of fully convective stars of given parameters (mass and chemical composition). The importance of this line lies in that it represents a borderline between an allowed region (on its left) and a forbidden region (on its right) for all stars with these parameters that are in hydrostatic equilibrium.

One can find a simple description of the Hayashi line by using a very crude model for fully convective stars. Assuming a constant adiabatic gradient ∇_{ad} throughout the star, we have a simple P-T relation for the whole interior:

$$P = C\,T^{1+n} \qquad \left(\nabla_{ad} \equiv 1 + \frac{1}{n}\right)$$

The constant C is related to the polytropic constant:

$$P = \frac{k_B \rho T}{\nu m_u} = K \rho^{1+\frac{1}{n}} \Rightarrow C = K^{-n} \left(\frac{k_B}{\nu m_u}\right)^{1+n}$$

For a given n, the solution of Lane-Emden equation results in (see pages 91 and 92):

$$K \sim \rho_c^{\frac{1}{3}} R^2 \sim M^{\frac{1}{3}} R$$

Thus:

$$C = C' R^{-3/2} M^{-\frac{1}{2}}$$

Where C' is known for given n and ν.

At the photosphere we therefore have:

$$\boxed{P_o = C' R^{-3/2} M^{-\frac{1}{2}} T_e^{n+1}} \quad *$$

P_o can also be found by integrating the hydrostatic equation through the atmosphere:

$$P_o = \int_R^\infty g\rho \, dr \simeq g_o \int_R^\infty \rho \, dr \quad , \quad g_o = \frac{GM}{R^2}$$

The integral $\int_R^\infty \rho \, dr$ is related to the optical depth by definition:

$$\tau \equiv \int_R^\infty \kappa \rho \, dr = \frac{2}{3}$$

$$\int_{R}^{\infty} \kappa \rho \, dr = \bar{\kappa} \int_{R}^{\infty} \rho \, dr \quad , \quad \bar{\kappa} = \kappa_0 P^a T^b$$

(radiative atmosphere with a simple absorption law)

Putting the pieces together, we have:

$$\boxed{P_0 = \text{Const.} \left(\frac{M}{R^2} T_e^{-b} \right)^{\frac{1}{a+1}}} \quad **$$

Equations * and ** can be written in logarithmic form: ($n = \frac{3}{2}$ is chosen)

$$\begin{cases} \log T_e = 0.4 \log P_0 + 0.4 \left(\frac{3}{2} \log R + \frac{1}{2} \log M - \log C' \right) \\ (a+1) \log P_0 = \log M - 2 \log R - b \log T_e + \text{Const.} \end{cases}$$

After using the $L \propto R^2 T_e^4$ relation, and eliminating $\log P_0$ between the above equations, we find:

$$\boxed{\log T_e = A \log L + B \log M + \text{Const.}} \qquad A = \frac{0.75a - 0.25}{b + 5.5a + 1.5} \; , \; B = \frac{0.5a + 1.5}{b + 5.5a + 1.5}$$

This represents a line in the H-R diagram, which is the "Hayashi Line".

As indicated by our discussion, the Hayashi line is far to the right in the H-R diagram, which means low T_e. Hence H^- absorption provides the dominant contribution to κ in the atmosphere, for which $a \simeq 1$, $b \simeq 3$.

This gives rise to:

$$\boxed{\log T_e = 0.05 \log L + 0.2 \log M + \text{Const.}}$$

We see the following facts:

(1) $\frac{\partial \log L}{\partial \log T_e} \gg 1 \Rightarrow$ Hayashi line is very steep, it is almost a vertical line.

(2) Hayashi line slightly shifted to the left with increasing M due to the $0.2 \log M$ term.

We reiterate that the Hayashi line is a borderline between an allowed region and a forbidden region for

for stars with given mass M and composition that are in <u>equilibrium</u>. In the allowed region (left) we will encounter a radiative region in the star before reaching the center. The larger T_e is, the earlier we reach this radiative region (recall that ∇_{rad} decreases with increasing T, due to the dependence of opacity on temperature, which signals tak over of radiation as the dominant source of energy transfer). In the forbidden region (right), the star is not in hydrostatic equilibrium.